EXTENDED OPPORTUNITY TUTORIAL 3

PROGRAMME : MECHANICAL ENGINEERING NATED 191

SUBJECT : MECHANOTECHNICS N5

TOPIC : ELEVATORS

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Instructions and information:

This tutorial covers the following concepts on elevators:

1. The car accelerating upwards
2. The car decelerating
3. Loss of potential energy and gain in kinetic energy

There are explanatory notes on how to solve problems involving elevators. You are encouraged to read and understand the meaning of terms so that you will be able to understand the circumstances described in questions.

NOTE**

- You are advised to read questions carefully and understand what you are required to DO
- You are advised to make a sketch of the problem in order to correctly place the forces that are acting on the body.
- You are advised to consistently practice solving different problems in order to maintain you muscle on solving them.
- Familiarise yourself with equations of motion and formulae applicable to problems regarding elevators.

#STAYFOCUSED

ELEVATORS

Elevator is a form of a vertical transport used primarily to move between multiple floors found in high rising buildings. These are used to transport both people and goods from one floor to another. There are two types of elevators namely passenger elevators and goods elevators. Figure 1 shows a typical passenger elevator.

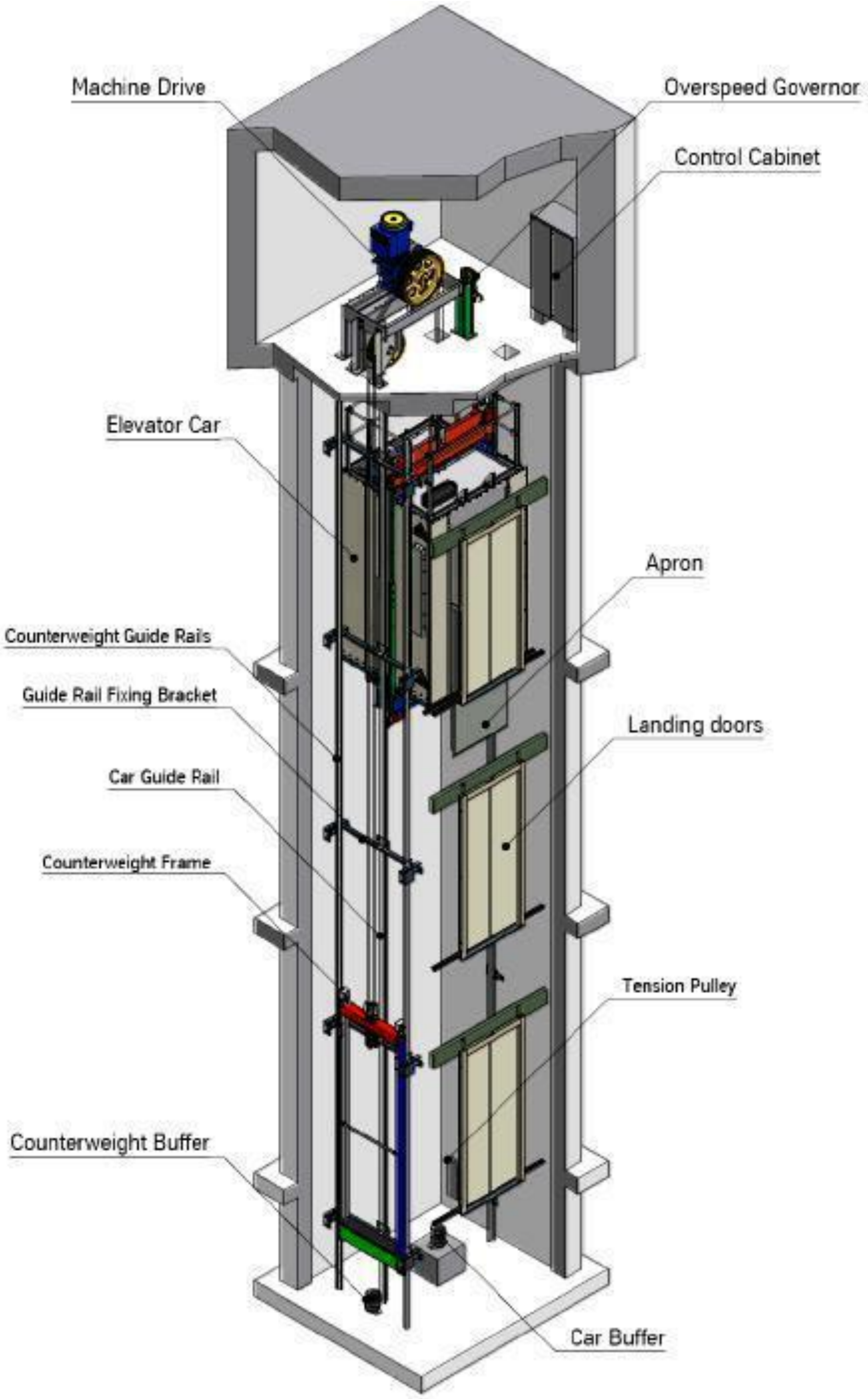

FIGURE 1: A typical passenger elevator

Symbols

m_1	=	mass of cage	kg
m_2	=	mass of balancing mass	kg
m	=	mass of drum	kg
T_1	=	Tight tension in rope	N
T_2	=	Slack tension in rope	N
F_μ	=	Force to overcome friction	Nm
T_μ	=	Torque on hoist to overcome bearing friction	Nm
S	=	Distance travelled	m
r	=	Radius of drum	m
k	=	Radius of gyration of drum	m
I	=	Moment of inertia of drum	kg m^2
α	=	Angular of acceleration of drum	rad / s^2
T	=	Torque	Nm

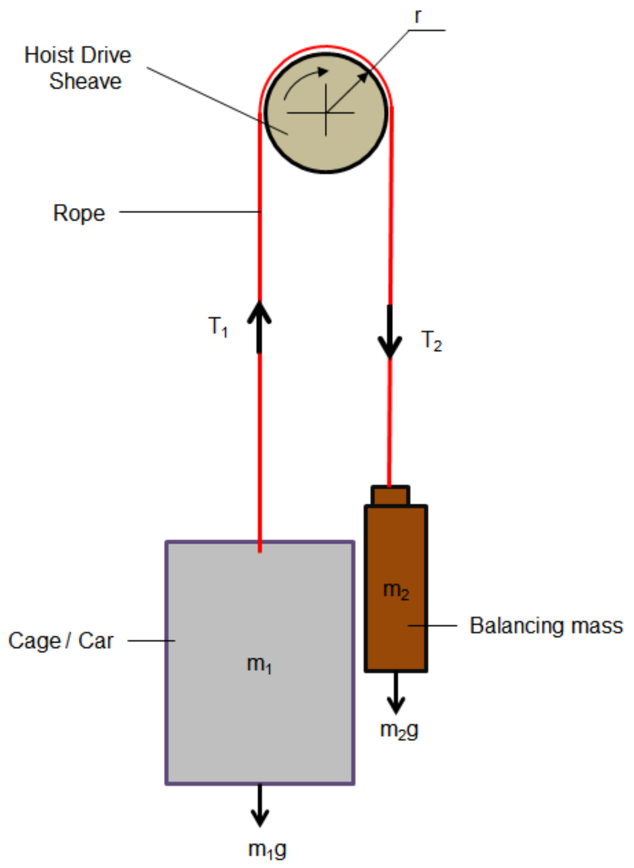

FIGURE 2: Schematic diagram of an elevator

Forces that act on the elevator system are:

1. Force due to mass of the car or cage = $m_1 g$
2. Force due to mass of the balancing mass = $m_2 g$
3. Force due to acceleration of the car or cage = $m_1 a$
4. Force due to acceleration of the balancing mass = $m_2 a$
5. Force to overcome friction = F_μ
6. Tight tension on the rope = T_1
7. Slack tension on the rope = T_2

As the cage is accelerating upwards, the balancing mass will be accelerating downwards. As the cage decelerates the balancing mass also decelerates. The following happens:

The cage accelerating upwards	Balancing mass accelerating downwards
$T_1 = m_1 g + m_1 a + F_\mu$ $= m_1 (g + a) + F_\mu$	$T_2 = m_2 g - m_2 a - F_\mu$ $= m_2 (g - a)\ \ F_\mu$
The cage decelerating	**Balancing mass decelerating**
$T_1 = m_1 g - m_1 a + F_\mu$ $= m_1 (g - a) + F_\mu$	$T_2 = m_2 g + m_2 a + F_\mu$ $= m_2 (g + a) - F_\mu$

Total torque to accelerate the elevator system will be the combined torque to:

- accelerate the loads (mass of the car + mass of occupants or goods)
- accelerate the drum due to moment of inertia
- accelerate within frictional force.

1. Torque to accelerate loads = $(T_1 – T_2) \times r$
2. Torque to accelerate the drum = $I \times \alpha$
   But also remember that; $I = mk^2$ AND $\alpha = a/r$
3. Torque within frictional force = T_μ

Therefore the TOTAL torque $\mathbf{T_{total} = (T_1 – T_2)\, r + I \times \alpha + T_\mu}$

Power required on the driving motor

$P_{motor} = T_{total} \times \omega$
$P_{motor} = T_{total} \times \frac{v}{r}$

remember that $\omega = \frac{v}{r}$

KINETIC ENERGY

As the elevator is moving the following kinetic energy E_k is gained:

1. The car kinetic energy = $\frac{1}{2}m_1v^2$
2. The balancing mass kinetic energy = $\frac{1}{2}m_2v^2$
3. Driving drum kinetic energy = $\frac{1}{2}I\omega^2$

$E_k total = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}(m_1v^2 + m_2v^2 + I\omega^2)$

$= \frac{1}{2}v^2\ (m_1 + m_2 + \frac{I}{r^2})$

POTENTIAL ENERGY

Potential energy $E_p = mgh$

Potential energy in the car = m_1gh
Potential energy in the balancing mass = m_2gh

Potential energy loss = $m_1\,g\,h - m_2\,g\,h$
$= g\,h\,(m_1 - m_2)$

It can be seen here that the gain in kinetic energy = loss in potential energy

Thus; $\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 = g\,h\,(m_1 - m_2)$

LOOKING ON THE DRUM

Torque on the drum T = Force x Radians turned
= Force x θ
Work done = T x θ
= T x $2\pi N$

Increase in kinetic energy = $\frac{1}{2}E_{k\ final} - \frac{1}{2}E_{k\ initial}$

$= \frac{1}{2}I(\omega_2)^2 - \frac{1}{2}I(\omega_1)^2$

$= \frac{1}{2}I[(\omega_2)^2 - (\omega_1)^2]$

Worked Problem

A cage of goods elevator carries a total load of 1 550 kg. The rope passes over a drum, 1 m in diameter and then to a balance mass of 450 kg. The mass of the rope, including balance mass ropes is 300 kg. The frictional torque on the drum shaft is 850 Nm. The maximum speed of the elevator is 6 m/s and it is accelerated to a maximum speed in 5 seconds.

Calculate the following:

1.1. The rope tensions T_1 and T_2 during acceleration

1.2. The torque to hold dead load

1.3. The torque to accelerate the rope

1.4. The total torque during acceleration

1.5. The motor power

Solution

Given:

$m_1 = 1550$ kg

$r = \frac{1}{2} = 0.5$

$m_2 = 450$ kg

$m_{rope} = 300$ kg

$T_\mu = 850$ Nm

$v = 6$ m/s

$t = 5$ sec

1.1

$$v = u + a$$

$$a = \frac{v - u}{t}$$

$$a = \frac{6 - 0}{5}$$

$$= 1.2 m/s^2$$

NOTE: T_μ given is not on the slides. It is only on the drum shaft hence it is applied only on the drum not affecting T_1 & T_2

$$T_1 = m_1 (g + a)$$
$$= 1550 (9.81 + 1.2)$$
$$= 17065.5 \text{ N}$$

$$T_2 = m_2 (g - a)$$
$$= 450 (9.81 - 1.2)$$
$$= 3874.5 \text{ N}$$

1.2

$$T_{load} = (T_1 - T_2) r$$
$$= (17065.5 - 3874.5)0.5$$
$$= 6595.5 \text{ Nm}$$

1.3 $T_{rope} = m_{rope} \times \alpha \times r$
$= 300 \times 1.2 \times 0.5$
$= 180$ Nm

1.4 $T_{total} = T_{load} + T_{drum} + T_{friction}$
$T_{total} = T_L + T_R + T_\mu$
$= 6595.5 + 180 + 850$
$= 7625.5$ Nm

1.5 Power $= T_{total} \times \omega$
$= T_{total} \times \frac{v}{r}$
$= 7625.5 \times \frac{6}{0.5}$

$= 91506$ W
$= 91.506$ kW

PROBLEM 1

The rope passes over a drum with a mass of 400 kg, a radius of gyration of 0,6 m and a diameter of 980 mm. The loaded cage of a goods elevator has a mass of 1 600 kg. The cage and balance mass of 850 kg move in guides. The frictional torque on the drum is 320 Nm. The frictional resistance between the guides and the cage is 620 N and between the guides and the balance mass is 480 N. The load has an upward velocity of 1,6 m/s and an upward acceleration of 0,6 m/s^2.

Calculate the following:

1.1. The total torque required on the drum

1.2. The power required at an instant when the load has an upward velocity of 1,8 m/s and an upward acceleration of 08 m/s^2

PROBLEM 2

A load of 600 kg is lifted by means of a rope which is wound around a drum and which then supports a balance mass of 200 kg. As the load is lifted, the balance mass is lowered. The drum has a mass of 10 kg and a diameter of 200 mm. If the load must be lifted 3 m and the system starts from rest, calculate the following:

2.1. The acceleration of the masses

2.2. Tight side tension T_1

2.3. Slack side tension T_2